647510 **0301207C1010** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination

June/July-2022 [OLD COURSE]

MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

1(A) Answer any one: **(07)**

(1) Derive an equation of tangent plane and normal to the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$at any point $(\alpha,\beta,\gamma)$of the sphere.

(2) Define: Right circular cylinder. Find equation of right circular cylinder with axis $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and radius r.

(B) Answer any one: **(04)**

(1) Find equation of right circular cylinder whose radius is 2 and axis is $\frac{x-1}{2}=\frac{y}{3}=z-3$.

(2) If the plane $-2x+2y-z=k$ touches the sphere $x^2+y^2+z^2+2y-6z+1=0$ then find the value of k. Also find the co-ordinates of point of contact.

(C) Answer any one: **(03)**

(1) Find the area of circle $x^2+y^2+z^2-8x+4y+8z-45=0, x-2y+2z-3=0$.

(2) Find the two tangent planes to the sphere $x^2+y^2+z^2-6x-4y-2z+5=0$ which are parallel to the plane $8x+y+4z=0$.

2 (A) Answer any one: **(07)**

(1) Define: Homogeneous function. If $u=x^3 f\left(\frac{y}{x},\frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=3u$.

(2) (i) For implicit function $f(x,y,z)=0$, obtain $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(ii) If $f(x,y)=0$ then obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(B) Answer any one: **(04)**

(1) If $f(x,y)=\frac{1}{\sqrt{y}}e^{\frac{-(x-a)^2}{4y}}$ then show that $f_{xy}=f_{yx}$.

(2) Define: Continuity of function of two variables and discuss the continuity of

$$f(x,y)=\frac{x^3+y^3}{x-y}\ ; x\neq 0$$
$$= 0 \ \ if \ \ x=y$$ at origin.

(C) Answer any one: **(03)**

(1) If $u=tan^{-1}\frac{x^3+y^3}{x-y}, x\neq y$ then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=sin2u.$$

(2) If $ye^{xy}=sinx$ then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at origin.

3 (A) Answer any one: **(07)**

(1) State and prove Taylor's theorem.

(2) Show that extreme value of the function $u=ax^2+by^2+cz^2$ with conditions $x^2+y^2+z^2=1$ and $lx+my+nz=0$ is given by

$$\frac{l^2}{a-u}+\frac{m^2}{b-u}+\frac{n^2}{c-u}=0.$$

(B) Answer any one: **(04)**

(1) Define: Approximate value of a function. If $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ then find approximate value of $f(1.9, 2.01, 4.8)$.

(2) Expand $logxy$ in powers of $x-1$ and $y-1$.

(C) Answer any one: **(03)**

(1) Find maxima and minima for the function $f(x,y) = x^2 - 2xy + \frac{y^3}{3} - 3y$.

(2) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$.

4 (A) Answer any one: **(07)**

(1) Define: Hermitian and skew Hermitian matrix. Prove that every square matrix can be expressed uniquely as the sum of a Hermitian and a skew Hermitian matrices.

(2) Define: Rank of a matrix, prove that the rank of the product of two matrices cannot exceed the rank of either matrix.

(B) Answer any one: **(04)**

(1) Define: Orthogonal matrix. If

$A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find the values of l, m and n.

(2) Find the rank of the matrix

$A = \begin{bmatrix} 1 & 3 & -3 & -1 & -4 \\ 1 & 4 & 1 & -2 & -2 \\ 2 & 9 & 0 & -5 & -2 \end{bmatrix}$ using echelon form.

(C) Answer any one: **(03)**

(1) Define: Inverse of a matrix. Find the inverse of

$A = \begin{bmatrix} 5 & 8 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ using elementary row operations.

(2) If $AB = BA$ and $S^2 = B$ then show that $(A^{-1}SA)^2 = B$.

5 (A) Answer any one: **(07)**

(1) State and prove Cayley-Hamilton theorem.

(2) Define: Eigen value and eigen vector of a matrix and prove that:

(i) There is unique eigen value with respect to any eigen vector of a given matrix.

(ii) There is more than one eigen vector with respect to any eigen value of a given matrix.

(B) Answer any one: **(04)**

(1) Using Cayley-Hamilton theorem find the inverse of

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

(2) Solve: $5x + 3y + 7z = 4,$

$3x + 26y + 2z = 9,$

$7x + 2y + 10z = 5$

(C) Answer any one: **(03)**

(1) Find eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

(2) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial.

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